# Are You Ready for AP Calculus?

Welcome! You have enrolled in Advanced Placement Calculus for the next school year. By enrolling in this course you are making a commitment to excellence in daily work. Successful students in AP Calculus exhibit the following characteristics:

- ➢ Daily review of new content material taught in class.
- ➢ Diligent completion of homework on a daily basis.
- ➢ Participation in study groups or working with a study buddy.
- ➢ Organizing notes and materials, practicing formulas and understanding major theorems and concepts as they are taught vs. cramming prior to a test.
- ➢ Asking questions in class and during Activity period.
- ➢ Perseverance in solving challenging problems.

The problems in this packet are designed to help you review topics from previous mathematics courses that are important to your success in AP Calculus. During the first week of classes in August an answer key will be available to you so that you may check your work. You will have opportunities during Activity Period and after school to ask questions. The packet will not be collected, but there will be a test on the material from the packet on each Friday until all Topics have been covered.

Enjoy your summer!
Dr. C. Williams
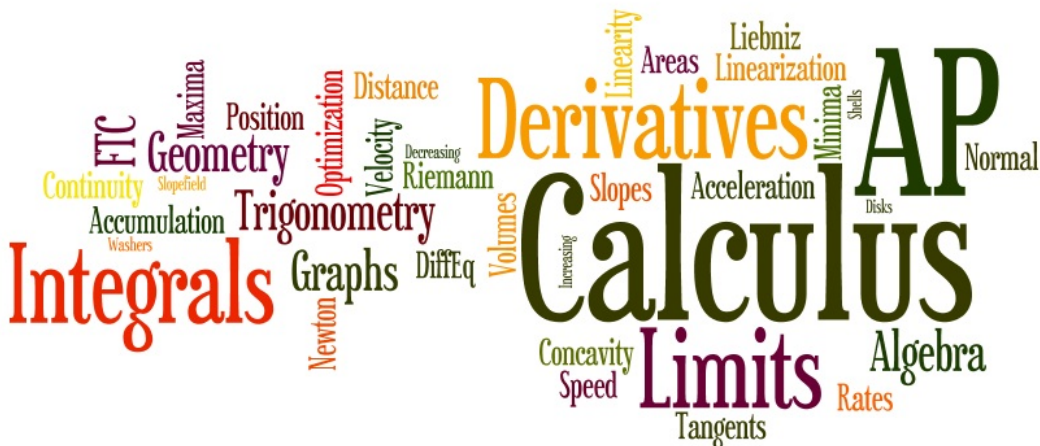
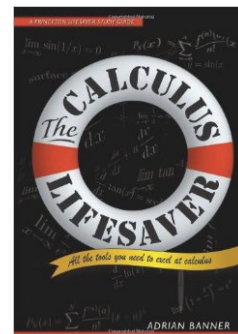1. Simplify each of the following expressions:

a) $\dfrac{x^3-9x}{x^2-7x+12}$

b) $\dfrac{x^2-2x-8}{x^3+x^2-2x}$

c) $\dfrac{x^3-8}{x-2}$

d) $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$

e) $\dfrac{9-x^{-2}}{3+x^{-1}}$

f) $\dfrac{2x}{x^2-6x+9}-\dfrac{1}{x+1}-\dfrac{8}{x^2-2x-3}=$

2. Rationalize each denominator:

a) $\dfrac{4}{1-\sqrt{5}}$

b) $\dfrac{2+\sqrt{3}}{1-\sqrt{3}}$

c) $\dfrac{5-\sqrt{x}}{3+\sqrt{x}}$

3. Complete the following identities:
   **These identities MUST be memorized for AP Calculus!

a) $sin^2x + cos^2x = $ _____

b) $1 + tan^2x = $ _____

c) $cot^2x + 1 = $ _____

d) $\dfrac{sinx}{cosx} = $ _____

e) $\dfrac{1}{cscx} = $ _____

f) $\dfrac{1}{cosx} = $ _____

g) $cos2x = $ _____

h) $sin\,2x = $ _____

IT'S A REOCCURRING NIGHTMARE, DOC...
I DREAM I'M IN THIS CLASS WHERE CALCULUS IS THE **EASY** PART!

4. Solve the following equations for the indicated variables:

a) $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ for a

b) $V = 2(ab + bc + ca)$ for a

c) $y^2 + 3yz - 8z - 4x = 0$ for z

d) $2x - 2yd = y + xd$ for d

e) $\dfrac{2x}{4\pi} + \dfrac{1-x}{2} = 0$ for x

f) $A = P + nrP$ for P

5. Factor each expression completely:

a) $x^6 - 64x^4$

b) $4x^3 - 8x^2 - 25x + 50$

c) $8x^3 + 27$

d) $81x^4 - 16$

6. Find all real solutions to each equation:

a) $27x^3 - 1 = 0$

b) $2x^3 - 6x^2 - 8x + 24 = 0$

c) $27^{2x} = 9^{x-2}$

d) $\log x + \log(x - 3) = 1$

e) $\log_{x-1} 64 = 3$

f) $\dfrac{3}{x} + \dfrac{x}{x+2} = -\dfrac{2}{x+2}$

g) $2x + 1 = \dfrac{5}{x+2}$

h) $|2x + 1| = x + 3$

7. Solve each of the following inequalities:
   a) $x^2 + 2x - 3 \le 0$ b) $2x^2 - 3x - 5 > 0$

   c) $\dfrac{2x-1}{3x-2} \ge 1$ d) $|3x + 2| < x - 4$

8. Find the remainder in each of the following division problems:
   a) $(x^5 - 4x^4 + x^3 - 7x + 1) \div (x + 2)$ b) $(x^5 - x^4 + x^3 + 2x^2 - x + 4) \div (x^3 + 1)$

9. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

10. Use long division to divide $(x^4 + 5x^3 + 6x^2 - x - 2) \div (x + 2)$

11. Use long division to divide $(6x^3 + 10x^2 + x + 8) \div (2x^2 + 1)$

12. Use synthetic division to divide $(5x^3 - 6x^2 + 8) \div (x - 4)$

13. Divide $(27x^3 - 64) \div (3x - 4)$

14. Using the Point-Slope form $y - y_1 = m(x - x_1)$,   write the equation of the line:
   a) With slope -2 through the point (4, 6)

   b) Containing the points (-1, 3) and (2, -4)

   c) Through the point (-1, 2) and perpendicular to the line $2x - 3y + 5 = 0$

   d) The line through (2, 3) and the midpoint of the line segment from (-1, 4) and (3, 2)

15. Find the equation of the circle with center at (1, 2) that passes through the point (-2, -1)

16. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$,
   a) Find the center and the radius

   b) Find the equation of the tangent line at the point (-2, 5)

17. If $f(x) = \{(3,5), (2,4), (1,7)\}$ $g(x) = \sqrt{x - 3}$

         $h(x) = \{(3,2), (4,3), (1,6)\}$ $k(x) = x^2 + 5$
   then determine each of the following:
   a) $(f + h)(1) =$ _____   b) $(k - g)(5) =$ _____

c) $(f \circ h)(3) =$ _____ d) $(g \circ k)(7) =$ _____

e) $f^{-1}(x) =$ _____ f) $k^{-1}(x) =$ _____

g) $(k \circ g)(x) =$ _____ h) $(kg)(x) =$ _____

18. Simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ where

a) $f(x) = 2x + 3$

b) $f(x) = \dfrac{2}{x-3}$

c) $f(x) = x^2 - 4x$

d) $f(x) = \sqrt{x+5}$

19. For each function, determine its domain and range:

a) $y = \sqrt{x^2 - 3x - 4}$

b) $y = (x - 3)^2$

c) $f(x) = \ln(x + 2)$

d) $f(x) = e^{x-5}$

20. Simplify:

a) $\cot x \sec x$

b) $\dfrac{\cos^2 x + \sin^2 x}{\cot x}$

c) $e^{1+\ln x}$

d) $\log_3 \dfrac{1}{81}$

e) $\ln \dfrac{1}{e}$

f) $\dfrac{4xy^{-2}}{12x^{-3}y^{-5}}$

21. **Without** a calculator, determine the exact value of each expression:

a) $\cos 0 =$ _____

b) $\sin \dfrac{3\pi}{2} =$ _____

c) $\sin \dfrac{7\pi}{4} =$ _____

d) $\cos \pi =$ _____

e) $\cos \dfrac{\pi}{3} =$ _____

f) $\cos \dfrac{3\pi}{4} =$ _____

g) $\tan \dfrac{5\pi}{4} =$ _____

h) $\tan \dfrac{2\pi}{3} =$ _____

i) $\sec \dfrac{\pi}{4} =$ _____

j) $\csc \dfrac{7\pi}{6} =$ _____

k) $\cot\frac{\pi}{2} =$ _____

l) $\arcsin\frac{1}{2} =$ _____

m) $\arctan 1 =$ _____

n) $\arccos\frac{\sqrt{2}}{2} =$ _____

22. Solve for all values of $0 \le x \le 2\pi$

a) $\sin 2x - \cos x = 0$

b) $\cos^2 x - \sin^2 x = 2\sin x$

23. Sketch each of the following functions:

a) $f(x) = \begin{cases} -x + 2, & x < 0 \\ 2x - 1, & x \ge 0 \end{cases}$

b) $f(x) = |x - 3| + 1$

24. After being dropped from the top of a tall building, the height of an object is described by $y = 400 - 16t^2$, where y is measured in feet and t is measured in seconds.

a) How many seconds did it take the object to reach the ground where $y = 0$?

b) How high is the projectile at $t = 2$, and approximately how fast is it falling?

25. If $f(x) = (2x - 1)(x + 5)(x - 3)$ then what are the x-intercepts of $f(x)$?

Find an example of an equation whose graph intercepts the x-axis only at $x = \frac{2}{3},\ -2\ and\ 1$

26. The equation $f(x) = (x + 1)(x - 1)(x^2 + 4)$ defines a _quartic function_ f. How many zeroes does f have? How many x-intercepts does the function $y = f(x)$ have?

27. Given that $y(t) = 128 - 16t^2$, find the value of the difference quotient $\frac{y(2+k)-y(2)}{k}$

28. Sketch the graph of $f(x) = \frac{x^2-1}{x-1}$. Find the domain and range of $f(x)$.

Describe the similarities and differences between the graph of $f(x)$ and the graph of $g(x) = x + 1$.

29. The quadratic function $f(x) = x^2 - 3x$.

a) Find the value of the difference quotient $\dfrac{f(3+h)-f(3)}{h}$.

b) Find an algebraic expression for the quotient $\dfrac{f(x+h)-f(x)}{h}$.

30. The point $P = (-3, 2.4)$ is on the ellipse $9x^2 + 25y^2 = 225$. Verify this and come as close as you can to finding the slope of the line that is tangent to the ellipse at point P.